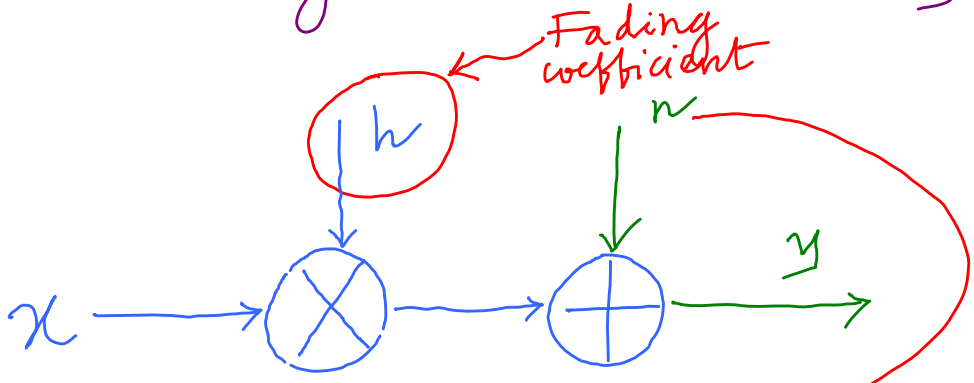
BER of wireless Comm. system:

wireless system $\Rightarrow$ Fading



White Gaussian noise
mean 0
variance $= \sigma^2$

Model for wireless:

$$y = h x + n$$

$h = a e^{j\phi}$

- y: Received symbol
- h: Fading coefficient
- x: Transmitted symbol
- n: noise

$$y = hx + n$$

Received power $= |h|^2 P$

$h = ae^{\phi}$ $\quad |h| = a$

$= a^2 P$

Fading SNR

$$SNR_F = \frac{a^2 P}{\sigma^2} = a^2 \frac{P}{\sigma^2} = a^2 SNR$$

From BER derived for BPSK modulation,

$$BER = Q\left(\sqrt{SNR_F}\right)$$

$$= Q\left(\sqrt{a^2 SNR}\right)$$

To find average BER, we have to average wrto the distribution of a Random Quantity

Depends on the Fading coefficient h

$$f_A(a) = 2ae^{-a^2}$$

Average BER,

$$= \int_0^{\infty} Q\left(\sqrt{a^2 SNR}\right) f_A(a)\, da$$

$$= \int_0^{\infty} Q\left(\sqrt{a^2 SNR}\right) 2ae^{-a^2}\, da$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

average BER for BPSK modulation in a Rayleigh fading channel.

Example: Compute the BER of a wireless communication system for $SNR_{dB} = 20$ dB.

$$10 \log_{10} SNR = 20$$

$$\Rightarrow \log_{10} SNR = 2$$

$$\Rightarrow SNR = 10^2 = 100.$$

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{100}{102}}\right) = \boxed{4.92 \times 10^{-3}}$$

Example: Compute SNR_{dB} required for a wireless system for $BER = 10^{-6}$.

$$10^{-6} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$\Rightarrow \quad \frac{SNR}{2+SNR} = \left(1 - 2\times10^{-6}\right)^2$$

$$\Rightarrow \quad SNR = \frac{2\times\left(1-2\times10^{-6}\right)^2}{1-\left(1-2\times10^{-6}\right)^2}$$

$$= 4.99\times10^{5}$$

$$SNR_{dB} = 10\log_{10} SNR = 56.98 \text{ dB}.$$

$$\approx 57 \text{ dB}.$$

For an AWGN or wireline channel, the corresponding SNR required to achieve BER $= 10^{-6}$ is 14.19 dB.

Therefore, for a wireless channel, the extra SNR in dB is,

$$\approx 57 - 14 = \boxed{43 \text{ dB}}$$

huge increase in required SNR to achieve same BER $= 10^{-6}$ in wireless channel.

Let us examine the reason for this.

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{1}{1+\frac{2}{SNR}}}\right)$$

$$= \frac{1}{2}\left(1 - \left(1+\frac{2}{SNR}\right)^{-1/2}\right)$$

$$= \frac{1}{2}\left(1 - \left(1 - \frac{1}{2}\cdot\frac{2}{SNR}\right)\right)$$

$$= \frac{1}{2}\left(\frac{1}{2}\cdot\frac{2}{SNR}\right) = \frac{1}{2\,SNR} \propto \frac{1}{SNR}$$

BER in wireless $\propto \frac{1}{2 \cdot SNR}$

in AWGN or wireline

$= Q\left(\sqrt{SNR}\right) = \frac{1}{2} e^{-\frac{1}{2} SNR}$

Decreasing exponentially w.r.t. SNR

Example: using formula for approximate BER, calculate SNR for BER $= 10^{-6}$ in wireless system.

$$10^{-6} = \frac{1}{2.5NR}$$

$$SNR = \frac{1}{2 \times 10^{-6}}$$

$$= 0.5 \times 10^{6}$$

$$SNR_{dB} = 10 \log_{10}(0.5 \times 10^{6})$$

$$= 56.98 \approx 57 \text{ dB}$$

BER of wireless system:

Exact: $BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$

Approximate: $BER = \frac{1}{2 \cdot SNR}$

At high SNR.